# Rule rather than Exception: Defeasible Probabilistic Dyadic Deontic Logic.

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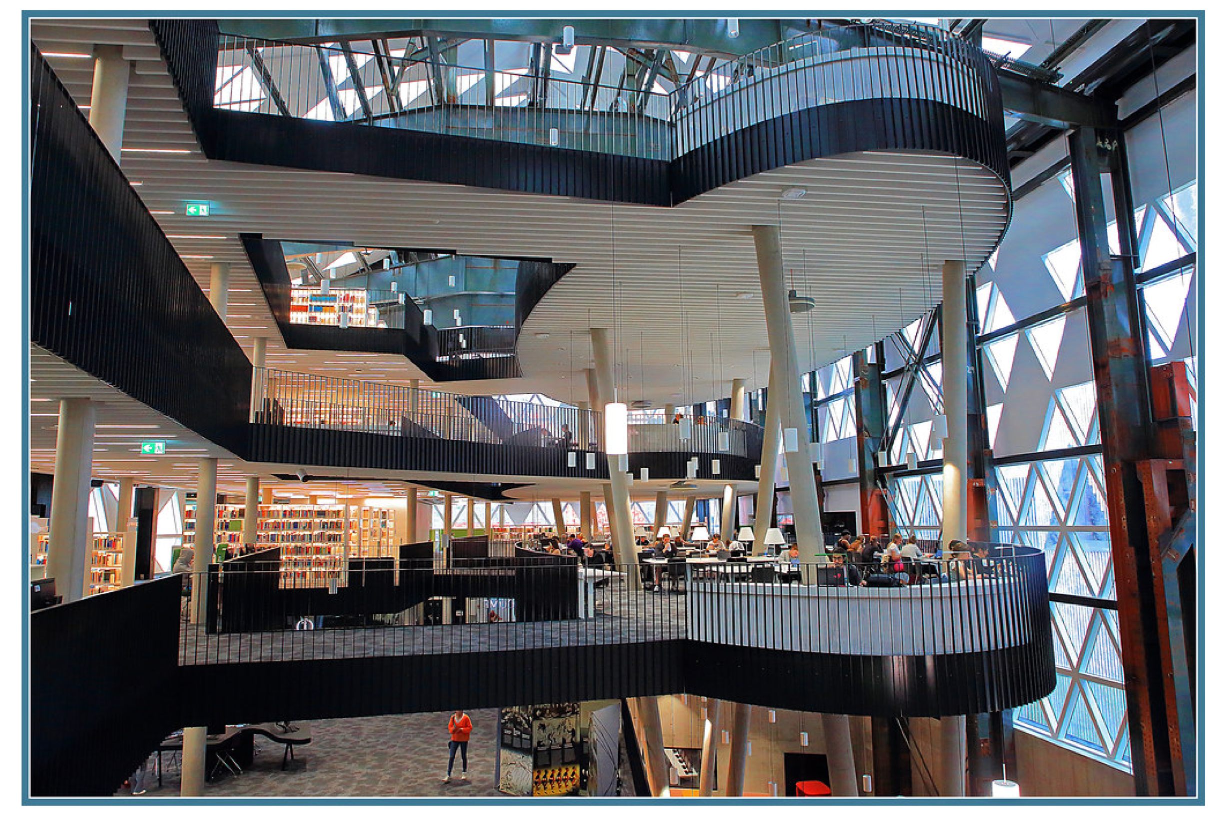
We take probabilistic deontic logic [de Wit et al., 2021]and make it defeasible using a method recently published in [Dong et al., 2022]. The specific probabilistic deontic logic we use is dyadic deontic logic [Hansson, 1969] combined with a multi-agent variant of probabilistic logic [Fagin and Halpern, 1994]. More specifically, we will use the developed method of specifying an upper and a lower bound logic that will define the strict and defeasible rules of an $ASPIC^+$ framework [Modgil and Prakken, 2014][Baroni et al., 2018]. The lower bound logic uses axiom system G of the Hansson-Lewis systems of Dyadic Deontic Logic combined with axioms for the probabilistic logic[Gabbay et al., 2013][Parent and van der Torre, 2018]. And for the upper bound logic, the axioms of Upward and Downward inheritance introduced in [Prakken and Sergot, 1996] are added.

Consider the following example scenario of an agent learning about the rules of a library. The agent does not know that it is a rule to be silent in a library, and attempts to derive such rules without breaking them or explicitly asking other agents about the rules. While in the library, the agent will discover that most people are silent in the library, though that there are exceptions –for example at the checkout counter–. Furthermore, the agent will encounter an ambiguous situation in which people talk inside a room. Multiple explanations are possible in this case: it is allowed to talk inside the room, the people do not know about the rule to be silent, or the people do not care about the rule i.e. they are breaking the rule.

We consider the described framework as a framework that is used by an agent to describe specific elements of its surroundings and reason about it. The framework combines multiple operators namely: strict and defeasible implications ($\rightarrow, \Rightarrow$), permissions and obligations ($O$, $P$), probabilistic and non-probabilistic formulas ($\alpha * w_i(\phi) \geq \beta$, $\phi$), theory of mind formulas ($\alpha_1 * w_i(\alpha_2 * w_j(\phi) \geq \beta_2) \geq \beta_1$) and also defeasible and non-defeasible knowledge. An important question therefore is: "What is the difference between a defeasible permission and an uncertain permission?" The difference is that defeasible knowledge is debunkable while the uncertainty about something is not debunkable, if the uncertainty is not defeasible in the first place. Normally $\phi$ is permitted; I am uncertain whether $\phi$ is permitted. Furthermore, we can express "normally I am uncertain whether $\phi$ is permitted." The work also opens up whether the framework should give preference to more certain information, i.e. $w_i(\phi) \geq 0.8$ versus $w_i(\psi) \geq 0.7$. There is more justification for $\phi$ than there is for $\psi$.



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## Research Question

- How to formalize reasoning about deriving norms from others their behaviour?

- Structured argumentation is used to make Probabilistic Dyadic Deontic Logic defeasible.
- We have multiple operators in this framework:
  - Strict($\rightarrow$) and Defeasible($\Rightarrow$)
  - Agent specific Probability ($w_i()$)
  - Permission ($P(\cdot \mid \cdot)$)and Obligation ($O(\cdot \mid \cdot)$)
- The probabilistic operator is indexed per agent $w_i(\phi)$: the uncertainty of $i$ about $\phi$.

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| | **Tautologies and Modus Ponens** |
| Taut. | All instances of propositional tautologies. |
| MP. | From $\theta$ and $\theta \rightarrow \delta$ infer $\delta$. |
| | **Reasoning with $\Box$** |
| $\Box$-K. | $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$ |
| $\Box$-T. | $\Box\phi \rightarrow \phi$ |
| $\Box$-5. | $\neg\Box\phi \rightarrow \Box\neg\Box\phi$ |
| $\Box$-Nec. | If $\vdash \phi$ then $\vdash \Box\phi$ |
| | **Reasoning with $O(-\vert-)$:** |
| COK. | $O(\chi \rightarrow \psi \vert \phi) \rightarrow (O(\chi\vert\phi) \rightarrow O(\psi\vert\phi))$ |
| id. | $O(\phi\vert\phi)$ |
| sh. | $O(\psi\vert(\phi \wedge \chi)) \rightarrow O((\chi \rightarrow \psi)\vert\phi)$ |
| Sp. | $P(\psi\vert\phi) \wedge O((\psi \rightarrow \chi)\vert\phi) \rightarrow O(\chi\vert(\phi \wedge \psi))$ |
| | **Interplay of $\Box$ and $O(-\vert-)$:** |
| Abs. | $O(\psi\vert\phi) \rightarrow \Box O(\psi\vert\phi)$ |
| Nec. | $\Box\psi \rightarrow O(\psi\vert\phi)$ |
| Ext. | $\Box(\phi \leftrightarrow \chi) \rightarrow (O(\psi\vert\phi) \leftrightarrow O(\psi\vert\chi))$ |
| $O$-$D^*$. | $\diamond\phi \rightarrow (O(\psi\vert\phi) \rightarrow P(\psi\vert\phi))$ |
| | **Reasoning about linear inequalities:** |
| I1. | $x \geq x$ (identity) |
| I2. | $(a_1x_1 + \cdots + a_kx_k \geq c) \leftrightarrow (a_1x_1 + \cdots + a_kx_k + 0x_{k+1} \geq c)$ (adding and deleting 0 terms) |
| I3. | $(a_1x_1+\cdots+a_kx_k \geq c) \rightarrow (a_{j_1}x_{j_1}+\cdots+a_{j_k}x_{j_k} \geq c)$, if $j_1, \ldots, j_k$ is a permutation of $1, \ldots, k$ (permutation) |
| I4. | $(a_1x_1 + \cdots + a_kx_k \geq c) \wedge (a'_1x_1 + \cdots + a'_kx_k \geq c') \rightarrow ((a_1 + a'_1)x_1 + \cdots + (a_k + a'_k)x_k \geq (c + c'))$ (addition of coefficients) |
| I5. | $(a_1x_1 + \cdots + a_kx_k \geq c) \leftrightarrow (da_1x_1 + \cdots + da_kx_k \geq dc)$ if $d > 0$ (multiplication of non-zero coefficients) |
| I6. | $(t \geq c) \vee (t \leq c)$ if $t$ is a term (dichotomy) |
| I7. | $(t \geq c) \rightarrow (t > d)$ if $t$ is a term and $c > d$ (monotonicity) |
| | **Reasoning about probabilities:** |
| W1. | $w_i(f) \geq 0$ (non-negativity). |
| W2. | $w_i(f \vee g) = w_i(f) + w_i(g)$, if $\neg(f \wedge g)$ is an instance of a classical propositional tautology (finite additivity). |
| W3. | $w_i(\top) = 1$ |
| P-Dis. | From $f \leftrightarrow g$ infer $w_i(f) = w_i(g)$ (probabilistic distributivity) |

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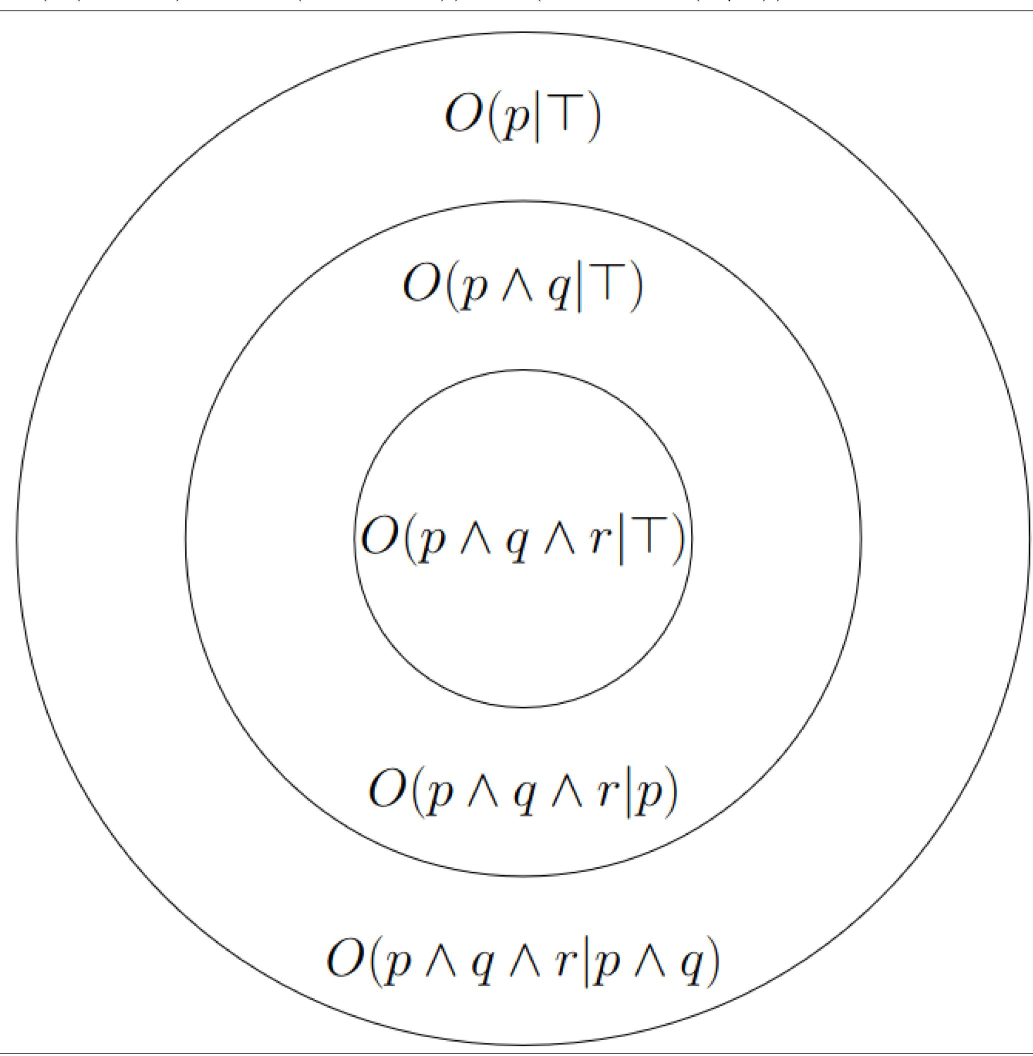
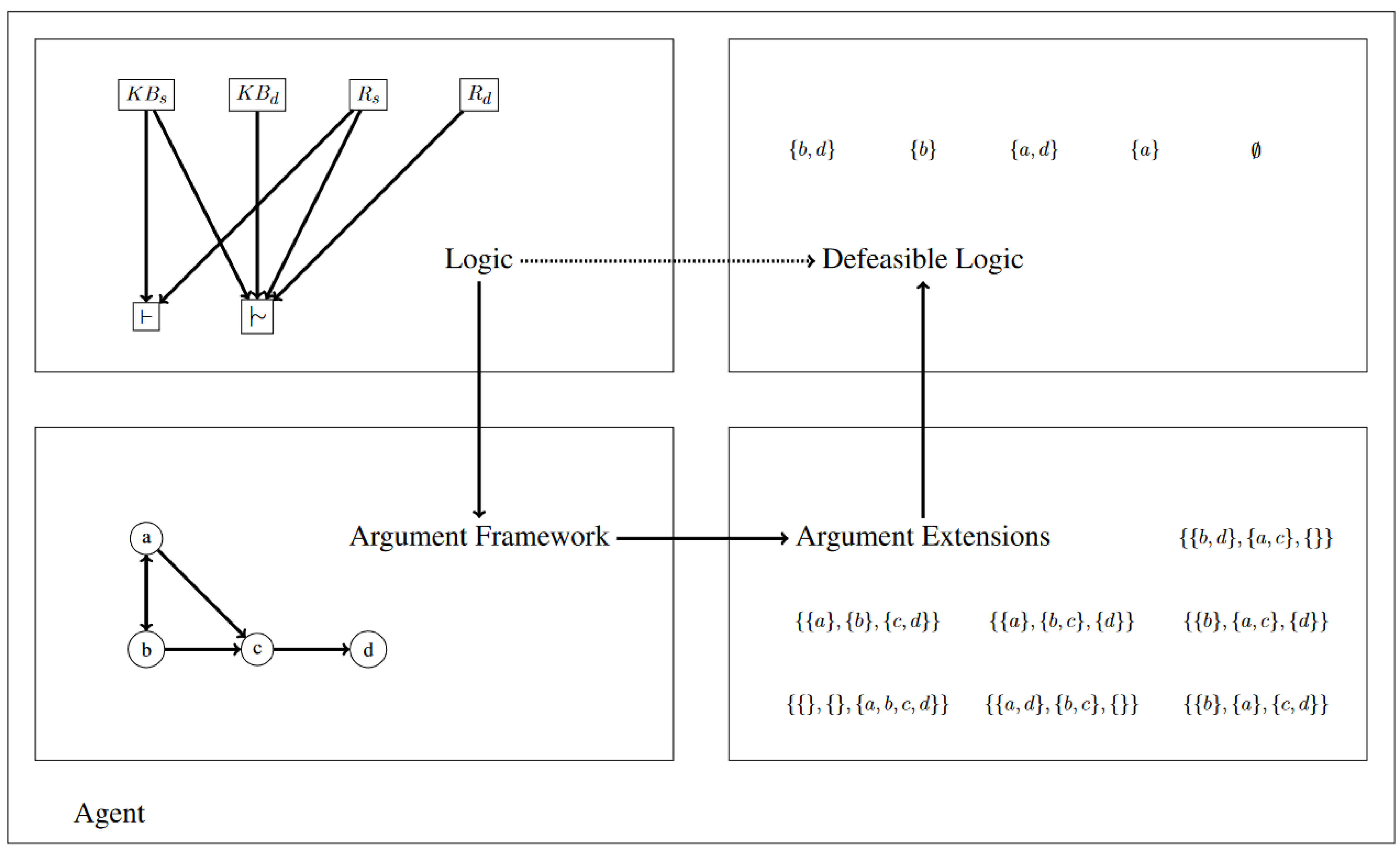
| Up. | $P\psi \rightarrow (O(\phi\vert\psi) \rightarrow O\phi)$ |
| Down. | $(\diamond(\phi \wedge \psi) \wedge \neg\Box(\neg\phi \rightarrow \psi)) \rightarrow (O\phi \rightarrow O(\phi\vert\psi))$ |




**Definition 1 (Language)** Let $\mathbb{P}$ be a set of atomic propositions and $Ag$ a set of agent. The language $\mathcal{L}$ of probabilistic DSDL is generated by the following two sentences of BNF (Backus Naur Form):

$$[\mathcal{L}_{DSDL}]\ \phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \Box\phi \mid O(\phi|\psi) \qquad p \in \mathbb{P}$$

$$[\mathcal{L}_{PDSDL}]\ f ::= \phi \mid \neg f \mid f \wedge g \mid \alpha_1 w_i(f_1) + \cdots + \alpha_n w_i(f_n) \geq \beta \qquad \alpha_j, \beta \in \mathbb{N}, \phi \in \mathcal{L}_{DSDL}, i \in Ag$$




Above is shown how the logic is made defeasible using Formal Argumentation. By first constructing arguments, then determining which arguments taken together are inconsistent using a binary relation. Then the sets are determined and lastly the content of the chosen arguments are collected in sets.

On the left, you can see the influence on the probability of multiple dyadic deontic formulas. The premise and conclusion of the operator influence the probability. With the size of the circle loosely representing the size of the probability.

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